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EURECOM

Part I

NFR: The Idea and a Basic Model

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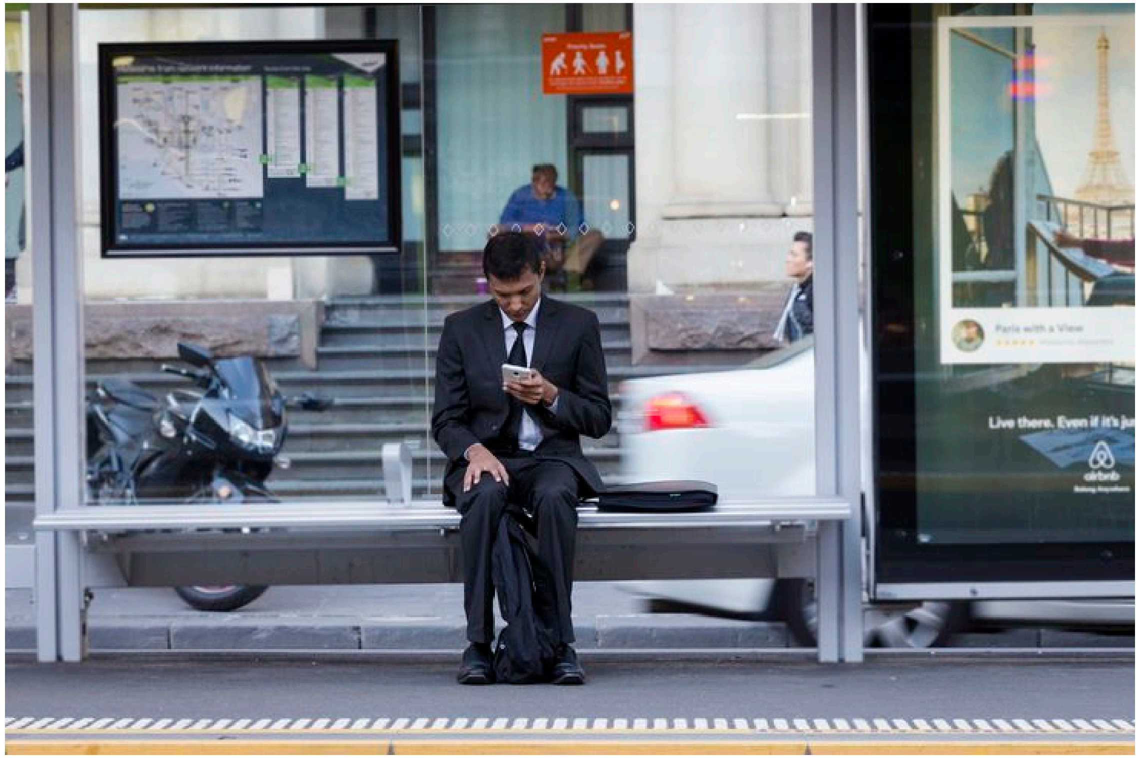
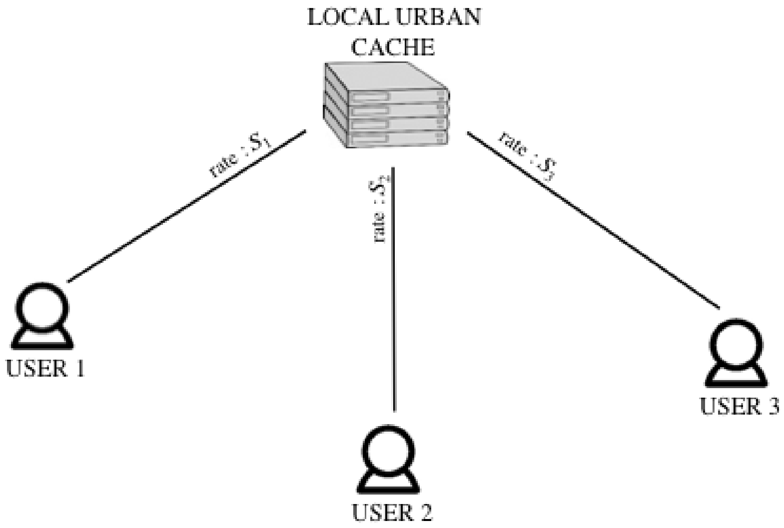
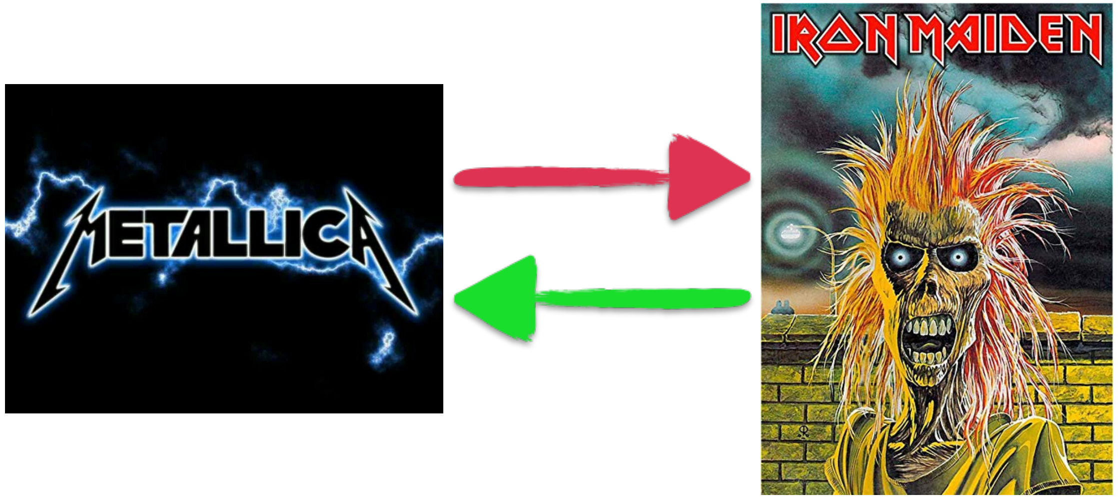
NFR: The Idea and a Basic Model

A Motivational Example





1. Alex is listening to music on his device during a **peak-traffic** time of the day.
2. He likes **Metal a lot**, and is a die hard fan of Metallica.
3. He is going to listen to **4-5 songs** while waiting.
4. He **hates interruptions** in the playback.



NFR: The Idea and a Basic Model

Bridging Two Entities

Content Library: $K = \{1, 2, \ldots, K\}$
Each file has a **probability** to be requested $p_0(i)$ (IRM).

MOST COMMON PRACTICE:
Sort the popularities and store the most popular, IT IS OPTIMAL.

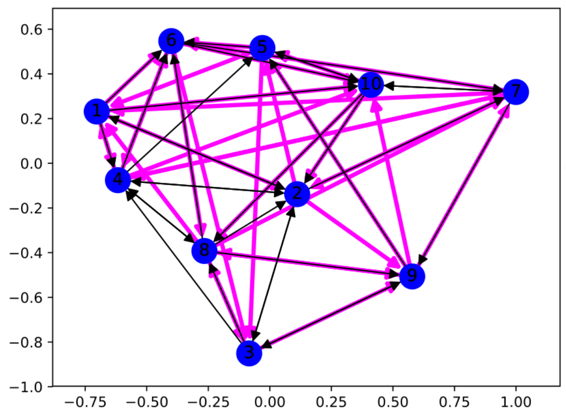
$p_0(1) > p_0(2) > \ldots > p_0(C) > p_0(C+1) > \ldots > p_0(K)$

FILE 1 **FILE 2** **FILE C**

A WIN-WIN Scenario

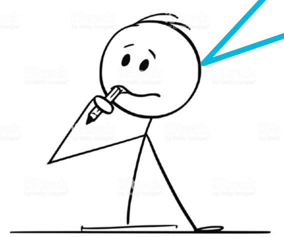
Caching has two clear benefits.
1. Reduced Backhaul Traffic.
2. Better Service for the users.

File i has a set of contents R_i with which it is considered to be **"similar/relavant"**



HOWEVER:
Caching is **effective**, **IF** the Cached files attract **MANY** requests.

Recommendations!



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Modeling the Sequential Content Consumption

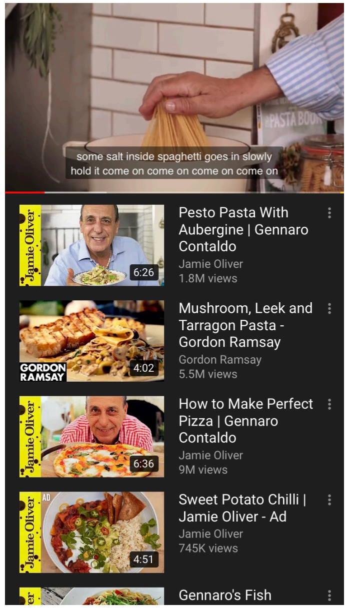
User actions

Log in to YouTube App… **what do we usually do?**

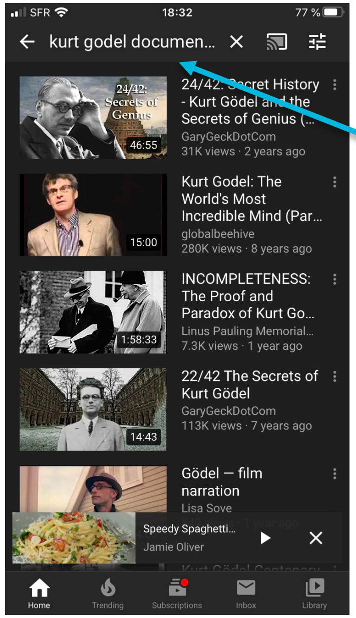
A. Recommendations.

B. Get bored, and look for something else.

Recommendations



Search Bar



Let's find out about the life of Gödel

Markovian Content Transitions

$$p_{ij} = \alpha \cdot \frac{r_{ij}}{N} + (1-\alpha) \cdot p_0(j)$$

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What is our Goal?

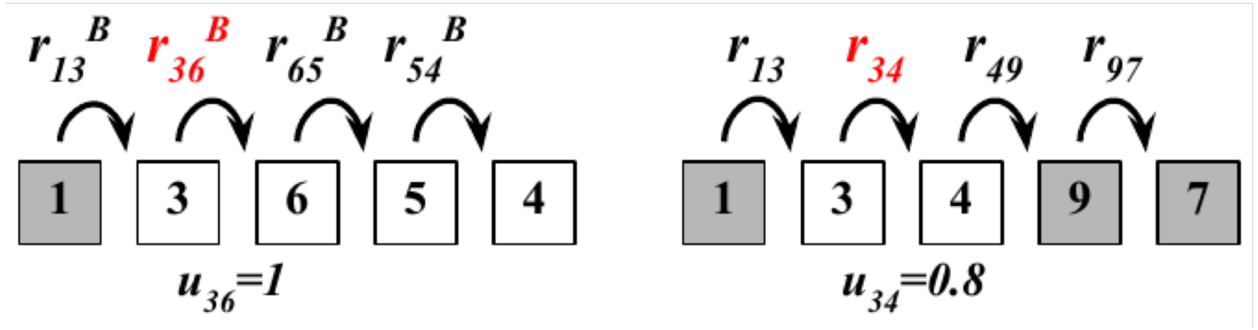
Minimize Network Cost for Long Sessions!

BUT HOW?

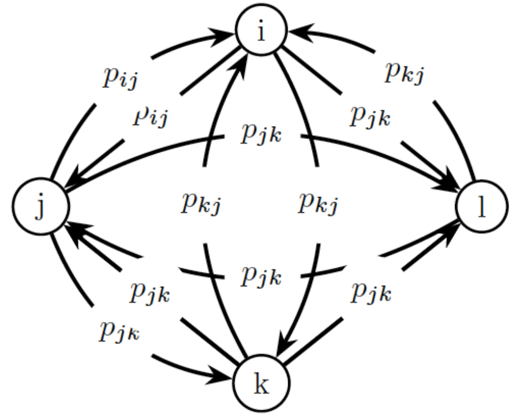
- **Modify baseline** recommendations and consider network cost as well.
- Decide **how often** *j* should be recommended after *i*, i.e., r_{ij}.

An example

One Cache Hit | **Three Cache Hits**



What about the "Long" of the Session?



π_1 π_2 π_3

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Formulation

Available expression for the steady state distribution

$$\underset{\mathbf{R}}{\text{minimize}} \quad \frac{\mathbf{p}_0^T \cdot (\mathbf{I} - \frac{\alpha}{N} \cdot \mathbf{R})^{-1} \cdot \mathbf{c}}{\frac{1}{1-\alpha}},$$

NONCOVNEX ☠

subject to

$$\sum_{j=1}^{K} r_{ij} \cdot u_{ij} \geq q \cdot q_i^{max}, \quad \forall i \in \mathcal{K},$$

Quality Constraint

$$\sum_{j=1}^{K} r_{ij} = N, \quad \forall i \in \mathcal{K}$$

Budget Constraint

$$0 \leq r_{ij} \leq 1 \quad (i \neq j), \quad r_{ii} = 0.$$

Probability Constraint

Constraints are Linear ✓

OK, now what?

Remove nonconvexity from the objective and insert it in the constraints.

$$\boldsymbol{\pi}^T = (1-\alpha)\mathbf{p}_0^T \cdot (\mathbf{I} - \frac{\alpha}{N}\mathbf{R})^{-1}$$

Objective now Linear

$$\underset{\mathbf{R},\boldsymbol{\pi}}{\text{minimize}} \sum_{i=1}^{K} \pi_i \cdot c_i$$

Nonconvexity in the constraints

$$\boldsymbol{\pi}^T = \boldsymbol{\pi}^T \cdot (\alpha \frac{\mathbf{R}}{N} + (1-\alpha)\mathbf{P}_0)$$

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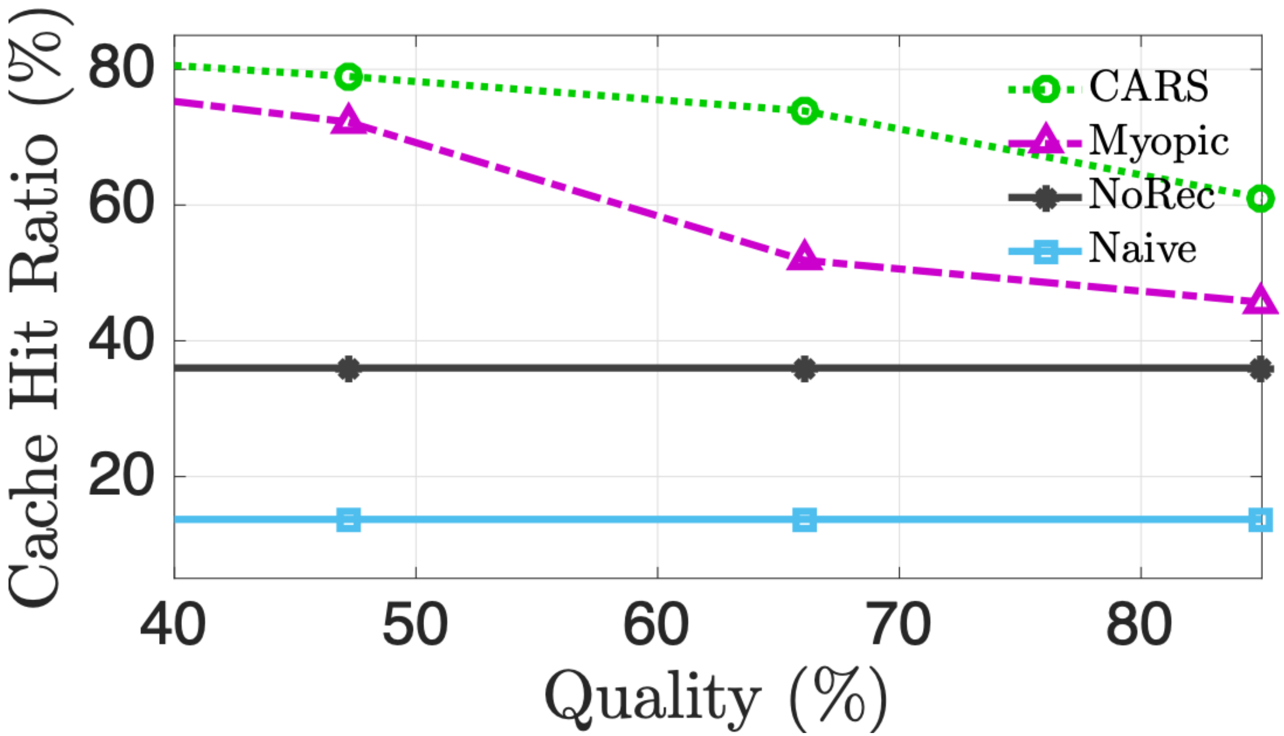
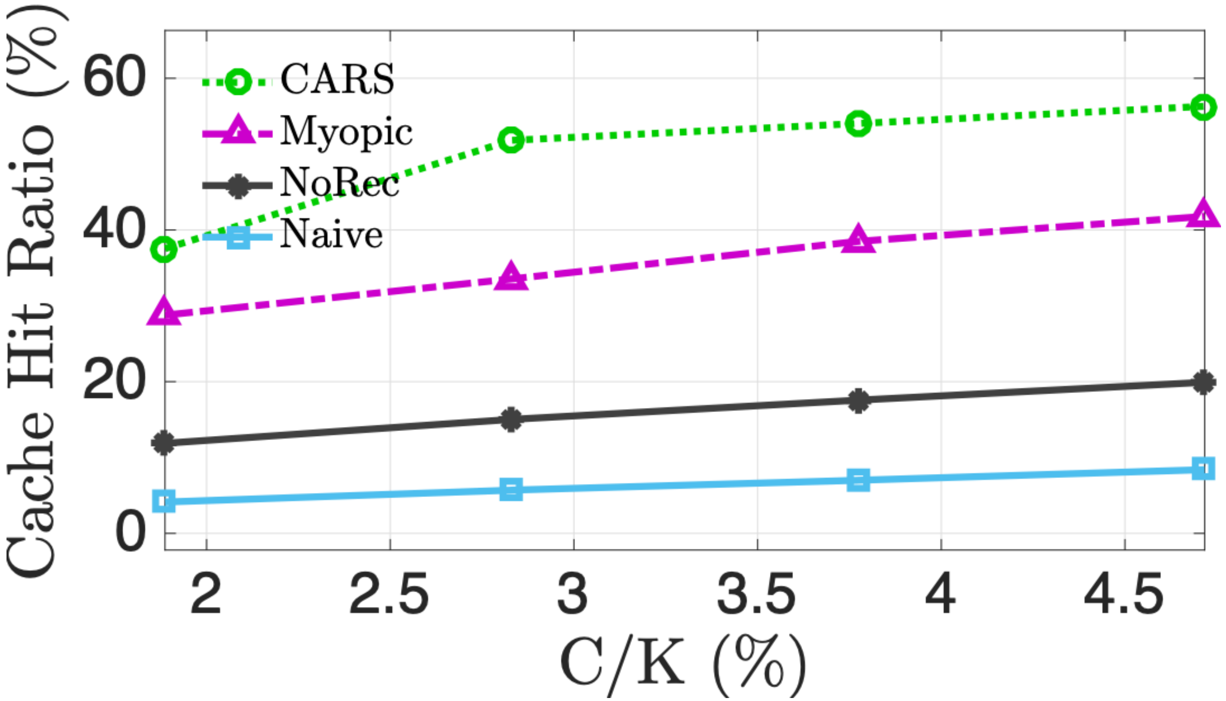
Solution and Results

Use ADMM!
Do an iteration for the one variable, then the other and approach the last inequality slowly.

Sounds good but keep in mind this is a heuristic.





Part II

NFR: Optimal Solution and an Extension

NFR: Optimal Solution and an Extension

Diving deeper…

Although it looks nasty, i.e., nonconvex, our problem has a special structure.

Observe…

$$\pi_i = \frac{\alpha}{N} \sum_{i=1}^{K} \underbrace{\pi_i \cdot r_{ij}}_{f_{ij}} + (1-\alpha) p_0(i)$$

Did this quadratic just become Linear?

Our initially nonconvex problem now became linear (trust me on that).

↓

But remember, we need to find r_{ij}. *This* f_{ij} is not useful.

↓

Solve the Linear Program for f_{ij}, and then compute r_{ij}. Is that possible?

MAIN RESULT

If all the values of p_0 are positive, then you can always go back to compute r_{ij}. Thus with this (reasonable) condition, our initial optimization problem can be solved in polynomial time as an LP.

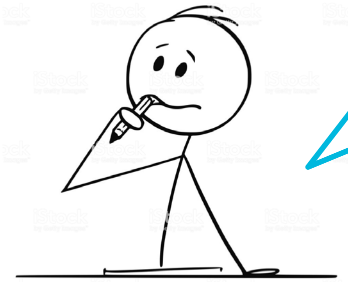
NFR: Optimal Solution and an Extension

Extending the Model

Earlier Assumption: The user clicks uniformly to one of the recommended items.

Not so accurate. Some positions are more favored than others. IMC 2010

Did we miss much by not considering this?



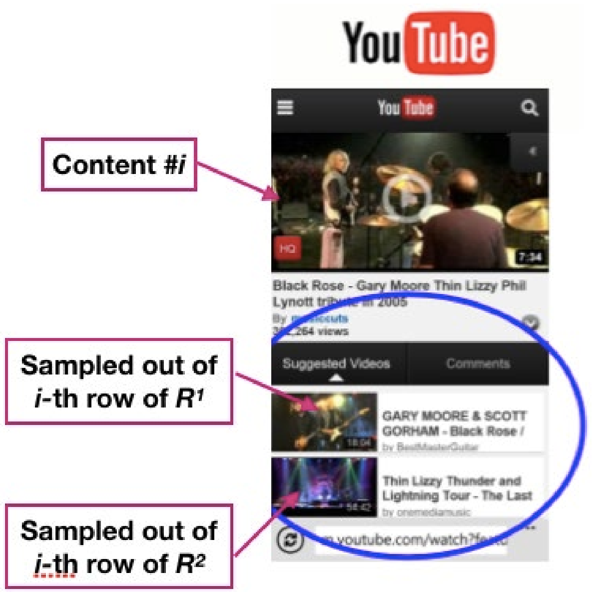
NFR: Optimal Solution and an Extension

Formulating the Problem

Reconsidering the transition probability of *i* —> *j*.
How many roads there are?

$$p_{ij} = \alpha \sum_{n=1}^{N} v_n \cdot r_{ij}^n + (1-\alpha)p_0(j)$$

We need one recommender matrix for every position.
It's the price we have to pay!



$$\underset{\mathbf{R}^1,\dots,\mathbf{R}^N}{\text{minimize}} \quad \mathbf{p}_0^T \cdot (\mathbf{I} - \alpha \cdot \sum_{n=1}^{N} v_n \cdot \mathbf{R}^n)^{-1} \cdot \mathbf{c}$$ → **NONCONVEX**

$$\text{subject to} \quad \sum_{j=1}^{K}\sum_{n=1}^{N} v_n \cdot r_{ij}^n \cdot u_{ij} \geq q \cdot q_i^{max}, \ \forall\, i \in \mathcal{K}$$ → **Quality (modified)**

$$\sum_{j=1}^{K} r_{ij}^n = 1, \ \forall\, i \in \mathcal{K} \ and \ n = 1, \dots, N$$ → **Stochasticity**

$$\sum_{n=1}^{N} r_{ij}^n \leq 1, \ \forall\, \{i,j\} \in \mathcal{K}$$ → **Implementable**

$$0 \leq r_{ij}^n \leq 1 \ \ (i \neq j), \ \ r_{ii}^n = 0 \ \forall\, i, \ n.$$ → **Probability**

Nonconvex - No Problem

Same trick as earlier applies, the problem can be transformed into an LP

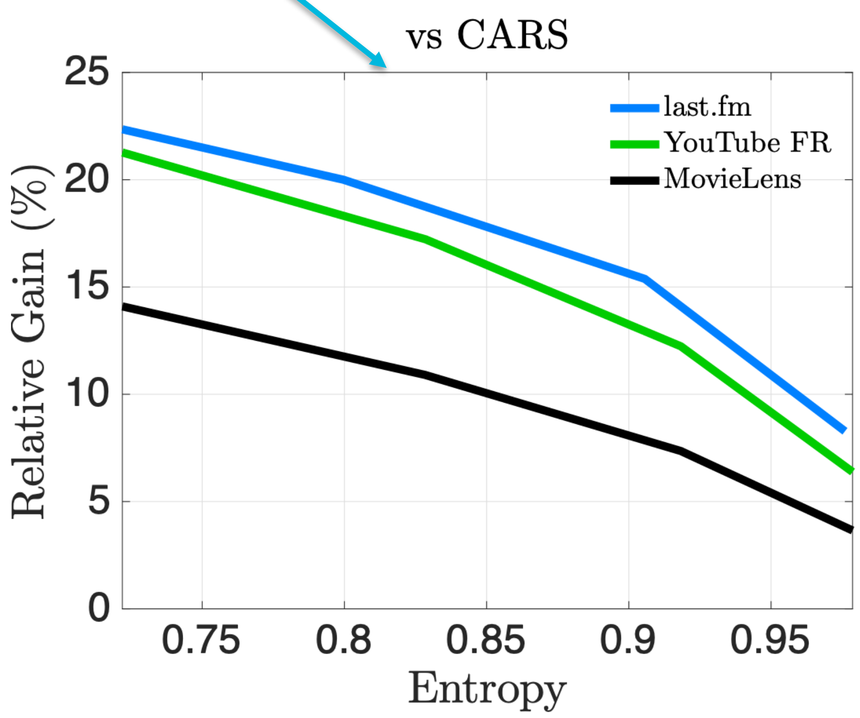
NFR: Optimal Solution and an Extension

Results

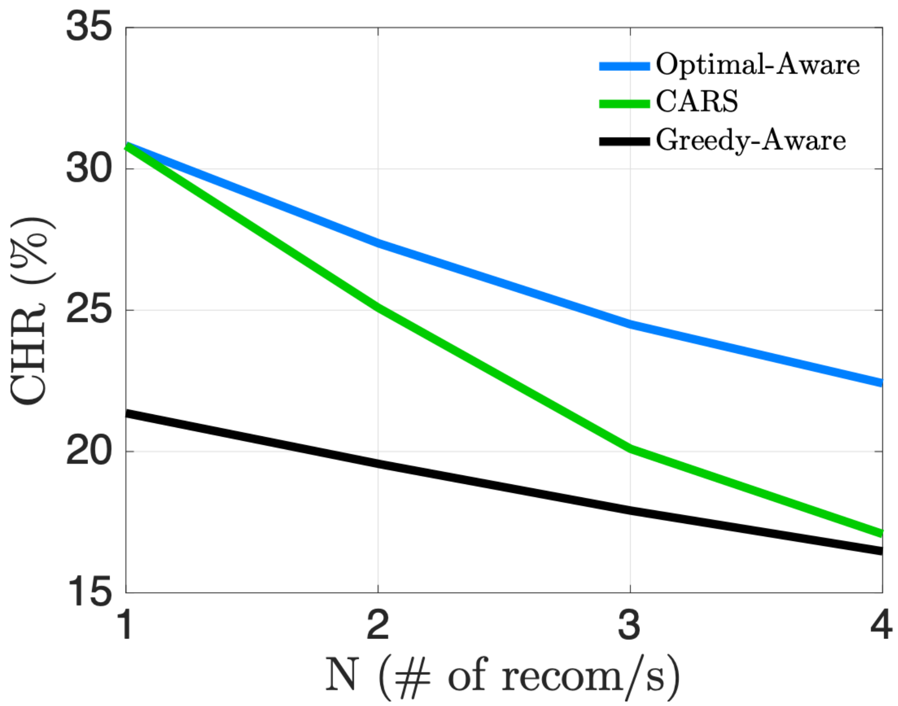
We can solve this challenging problem optimally…
But ok, what do we gain in return?

- **Users who click randomly have HIGH entropy.**
- **Users who click deterministically have LOW entropy.**

Look-Ahead strategy ignoring position importance



Lower entropy —> Higher Gains



Low Entropy with many recommendations can make a big difference

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THANK YOU FOR YOUR TIME

Any questions?